

Challenges in Experimental Vibration Analysis for Structural Identification and Corresponding Engineering Strategies

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ABSTRACT: Structural identification (St-Id) of long-span bridges by vibration testing under operation provides a starting point for quantitatively characterizing the in-service mechanical characteristics and behaviours of these complex constructed systems by designing and integrating additional experiments such as local NDE and crawl-speed load tests. The resulting characterization can serve as an effective baseline for structural health monitoring, designing more reliable and cost-effective structural retrofits, or developing and implementing more timely and efficient maintenance procedures. However, there are various uncertainties involved in the experimental and identification processes that impact the reliability of St-Id especially if vibration testing is the only experiment, these serve as a barrier to more widespread applications in civil engineering practice. The prevailing excitations (wind and traffic), environmental conditions (radiation and ambient temperature), experimental hardware (sensors, cabling and data acquisition system), and the execution of the experiment (array density and distribution, data acquisition parameters, on-site quality control, etc) all have a significant influence on the field test data quality and whether this data can be reliably processed for dynamic characteristics. Analytical modelling of complex structural systems and their components for St-Id brings its own significant uncertainties. Recognizing the impact of various uncertainty mechanisms and taking appropriate measures to quantify, bound and mitigate their impacts will greatly benefit bridge owners and engineers.

A vibration test on a long-span suspension bridge is leveraged as an example to illustrate a number of possible strategies for coping with the challenges presented above for practical identification of the structural dynamic characteristics of large-scale constructed systems. The design and implementation of a multi-reference field test are first presented to illustrate how uncertainties can be mitigated from an experimental point of view. Next, data pre-processing strategies including data inspection, time window selection, band-pass filtering, averaging and windowing are proposed to reduce data errors and to detect possible causes of outliers in the measured data (higher traffic, higher wind, temperature shocks, construction activity, unusual activity, etc). Three separate St-Id post-processing methods, including Peak-Picking, PolyMAX, and Complex Mode Indicator Function (CMIF), are applied for accurate structural modal parameter identification. Statistical analysis of the St-Id results from a number of time history windows are also performed, providing effective ways to investigate window relevance, data reliability and how they affect St-Id results. The identification results obtained for both the bridge spans and the towers demonstrate that the demonstrated field testing and data processing methods may provide a reliable bridge characterization.

KEYWORDS: Structural Identification, Ambient Vibration, Uncertainty and Mitigation, Signal Processing

1. INTRODUCTION

Structural identification (St-Id), which refers to any systematic approach for identifying structural parameters through the use of input and/or output test data, is a key enabler for structural health monitoring applications (Catbas et al. 2000, Zhang 2006). Following thirty years of development after the first article on St-Id (Hart and Yao 1977), applications in engineering practice have advanced the state of the art and developed St-Id into an effective tool for improving the reliability of bridge operational and maintenance management. Examples of the St-Id of many different bridge structural systems have been published in the literature including the identification of concrete box girder bridges (Peeters and DeRoeck, 2001), steel arch bridges (Grimmelsman et al. 2008) and long span suspension bridges (Xu et al, 1997; Ko et al, 2002; Grimmelsman et al., 2008; Siringoringo and Fujino, 2008; Pakzad and Fenves 2009). These practical implementations combining vibration tests and various data processing approaches advanced the state-of-the-art of St-Id, and illustrate the significant potential of St-Id for practical engineering applications.

Although there are many St-Id examples available in the literature, significant uncertainties involved in vibration testing and its interpretation still pose major challenges to reliable St-Id that is complete, accurate and robust, and hinder a more routine adoption of St-Id approaches in support of bridge operational and maintenance management decisions. The following uncertainties are often present in vibration tests, structural modelling, and signal processing procedures, and may lead to unreliable St-Id results: (1) prevailing excitations (wind and traffic) and environmental conditions (humidity, wind and most important, temperature) impart uncertainties into ambient or forced- vibration experiments; (2) frequently unavoidable measurement noise and bias errors arise from the deployed experimental hardware components (sensors, cabling and data acquisition system) and experiment design (array density and distribution, data acquisition parameters, on-site quality control, etc); (3) real structures are not accurately and completely modelled in the St-Id process and even considered in the finite element updating process, especially in relation to complex structural components like cables, bearings, soil-pile-foundation interaction, and tower-superstructure interfaces; (4) various data pre-processing methods with pre-determined parameters for data filtering, re-sampling, windowing and averaging produce different forms of “cleaned” data, and various post-processing methods may produce different identification results, depending on their capabilities to capture the signal characteristics in a noisy environment.

These uncertainties have a direct impact on the credibility of St-Id results and obstruct the St-Id concept's further development and applications. A number of researchers have recognized the importance of investigating and scientifically characterizing the uncertainties in St-Id and have proposed various approaches to deal with these uncertainties for more reliable St-Id results (Grimmelsman and Aktan 2005). Moon and Aktan (2006) conducted a detailed review of the impact of uncertainty on the structural identification of constructed systems. In their review, the unique attributes of constructed systems observed during St-ID studies were classified as either aleatory uncertainty (including heterogeneity, sensor/cable noise, etc.) or epistemic uncertainty (unknown or less understood structural behaviours). Peeters and DeRoeck (2001) studied variations of eigenfrequencies as a function of temperature by using one year of data from the Z24-Bridge measurement program. It was observed that the frequencies decreased as temperature increased. Nagayama et al. (2008) identified structural modal characteristics from ambient vibration data for a long-span bridge, and studied their dependencies on the wind velocity and response amplitude level. They found that natural frequencies decrease and damping ratios increase as the wind velocity increases, especially in the low order modes. Reynders et al. (2008) developed a method to bound modal parameter uncertainty by using the reference-based covariance-driven stochastic subspace identification (SSI) algorithm, in which the variance errors were estimated using the first-order sensitivity of the modal parameter estimates to perturbations of the measured output-only data. Omenzetter et al. (2004) proposed a multivariate outlier detection method to automatically filter out the outliers due to unusual events (like sudden settlement of foundation, ground movement, excessive traffic load or failure of post-tensioning cables). Catbas et al.

(2008) investigated how to minimize the uncertainties related to phenomena which are difficult to model in structural monitoring by interpreting one-year of monitoring data from a long span truss bridge through a structural reliability method. Pakzad and Fenves (2009) conducted the statistical analysis of the St-Id results for the Golden Bridge from different time windows to give the distributions of the structural modal parameters and their 95% confidence intervals. Ni et al. (2005) used a novel data processing technique called “Support Vector Machine” to model the temperature effects on the modal frequencies of a cable-stayed bridge. A Bayesian framework was presented by Beck and Katafygiotis (1998) to explicitly incorporate the uncertainties for structural parameter identification. Recognizing the challenges rising from uncertainties in vibration testing and data processing, and the importance of the St-Id method for structural health monitoring and infrastructure asset management, ASCE-SEI has re-established the St-Id of Constructed Facilities Committee (ASCE 2009). A state-of-the-art report detailing available approaches, methods and technologies for effective practice of St-Id of constructed systems has been prepared by this committee. The report also discusses the uncertainties involved in each stage of the St-Id process. The works reviewed above all contributed to advancing the state-of-the-art for coping with uncertainties from experimental, analytical, and data interpretation stages in the St-Id process. The research presented in this article is a further investigation of efficiently transforming response measurements from a structure into estimates of structural parameters for many practical uses.

1.1. Objective and Scope

The objective of this paper is to investigate a methodology for experimental characterization of long-span bridges by ambient vibration testing, with a particular focus on data quality evaluation, uncertainty reduction, and reliable structural identification. The various aspects of the experimental characterization, including field testing and data collection, signal pre-processing to clean the data, and signal post-processing to identify structural parameters of a long span suspension bridge are presented as an example to demonstrate the proposed methodology. A multiple-reference measurement scheme was first designed and implemented for this bridge to measure vibrations simultaneously in each of the principal response directions (vertical, lateral, longitudinal and torsional). It included thirty “stationary” accelerometers that were distributed throughout the bridge spans and were monitored during the entire field testing program, and fifteen “roving” accelerometers that were temporarily installed in specific regions of the bridge (north side span, south side span, main span, north tower and south tower) along with the stationary accelerometers. Significant care was taken for the sensor installation and on-site quality control measures were implemented to reduce data uncertainty during the experiment stage. A follow-up study focused on pre-processing the recorded structural responses, and included procedures related to data inspection, time window selection, digital filtering, exponential windowing, and time domain or frequency domain averaging for uncertainty mitigation. Cross correlation functions, frequency response functions, and complex mode indicator functions (CMIF) of different data sets were compared to study data relevance and reliability. Following this, three data post-processing methods including Peak-Picking, PolyMAX and CMIF were executed independently to identify the structural dynamic characteristics. Correlation studies of the St-Id results from these separate methods verified that the identified bridge parameters are reliable and gave the bridge owner confidence to reference these results for bridge rehabilitation decisions.

2. AMBIENT VIBRATION TEST

2.1 Bridge Description

An ambient vibration test of a long-span suspension bridge (**Fig. 1**) was recently performed to identify its dynamic characteristics. The main span of the bridge is 549 m long, with an anchorage to anchorage total length of 886 m. This bridge was commissioned on January 1961, and is carrying approximately 105,000 vehicles each day. The suspension bridge was designed with an 8.5-meter-deep stiffening truss under the deck, allowing wind to flow through the bridge. Six lanes of vehicular traffic rest on a series of laterally arranged transverse floor trusses, and these transverse trusses are framed into two longitudinal stiffening trusses within the same planes of main cables. A system of lateral stiffening trusses with the longitudinal and transverse trusses forms a rigid frame to resist wind and other loads. Two main cables with 976 m lengths support the main span and two side spans. Structural safety



Fig.1 The suspension bridge investigated

inspection and rehabilitation have been performed several times on the bridge since 1980, which included replacing the roadway decks, repairing the structural steel, modifying expansion joints, replacing existing rocker bearings, improving the drainage system, re-wrapping the main cables, and rehabilitating the electrical systems.

A vibration-based structural identification program was conducted to estimate the dynamic characteristics and related earthquake vulnerability of the bridge. In general, there are three possible experimental approaches that can be used to measure and identify the dynamic characteristics of an in-service constructed structure: (1) controlled forced excitation testing in which the external dynamic excitation supplied to the structure is provided by a mechanical excitation device such as a linear or eccentric mass shaker; (2) random vibration testing utilizing the natural and service live loads (wind, traffic, etc.) acting on the structure; and (3) free vibration measurements after initial conditions are imposed on the structure. Random vibration testing via the assumed random natural and service live loads already acting on the structure is by far the most practical and commonly used method for identifying the dynamic characteristics of long-span bridge structures. This approach is often referred to as ambient vibration testing or operational modal analysis, and is the method used in this study.

2.2 Sensors, Cables and Data Acquisition

The acceleration responses of the bridge were measured using Model 393C seismic accelerometers from PCB Piezotronics, Inc. This accelerometer utilizes the piezoelectric effect of quartz to convert acceleration directly into a low impedance voltage signal. The frequency range for the accelerometer is 0.025 Hz to 800 Hz, which far exceeds the typical frequency range of interest for dynamic characterization of a suspension bridge. It was determined from the accelerometer manufacturer's specifications that long cable lengths would not adversely affect the accuracy of the measurements for the low frequency band that was relevant to the ambient vibration testing of the bridge (< 10 Hz). This was an important consideration in designing the instrumentation for the field testing because many of the accelerometers would be distributed over very long distances from a central data acquisition station. It should be noted that although various wireless sensor systems have been promoted, their performance reliability especially in relation to synchronization and electromagnetic interference remain unproven.

The data acquisition hardware components were placed inside a watertight cabinet located behind a traffic barrier near the middle of the main span of the bridge. This location was selected as it resulted in the most efficient sensor cable layout for all of the individual test setups. A temporary main power service was installed at the cabinet location for use during the vibration testing. A cabling scheme was developed to further simplify the field installation of the sensors as much as possible, and utilized a number of 25.4 m lengths of multiple conductor twisted pair cables with breakout boxes installed at each end. These cables were routed along a maintenance walkway from data acquisition cabinet to the north and south bridge towers. A single coaxial cable was routed from each accelerometer to the breakout boxes at the towers. The cables from some of the accelerometers located on the stiffening trusses in the main span were routed directly to the data acquisition cabinet.

The data acquisition system used in the ambient vibration test consisted of an HBM MGCPlus data acquisition chassis with ML801B voltage modules and a laptop computer with data acquisition

software. The analog to digital conversion of the acceleration measurements was accomplished by Model ML801B voltage modules. Each voltage module is an 8 channel, 2.4 ksamples/s digitizer. These modules gave the data acquisition system the capability to sample a large number of channels simultaneously and at very high speeds. The data acquisition input modules provide several filtering options including both low and high pass filters. Bessel or Butterworth type filters can be used. The default filtering is an anti-alias filter that is applied at 10% of the total sampling rate. A sampling rate of 200 Hz was selected for the bridge vibration measurements. Therefore, the anti-aliasing filter was applied at 20Hz in the field test. A laptop computer connected to the HBM system was used to control the data acquisition system and to store the measurement data. The data acquisition system was operated automatically throughout the duration of the data collection.

2.3 Instrumentation Scheme

A roving instrumentation scheme was utilized in the majority of previous ambient vibration tests of long-span bridges that were described in the available literature. In a roving instrumentation scheme, a limited number of accelerometers are roved to different locations on the bridge using a series of test setups while a similarly small number of accelerometers remain in stationary locations on the structure. The latter sensors are commonly referred to as “reference” accelerometers, and their measured data serve as a baseline for linking the data acquired from the various roving locations together. Both the roving and the reference accelerometers are typically arranged on the structure in such a manner that the vertical, longitudinal, torsional and lateral components of the vibration response at each measurement station can be recorded. This can be accomplished with a minimum of three accelerometers (two vertical and one lateral for the span motions, or two longitudinal and one lateral for the towers). In many cases, as few as 6 sensors (3 roving and 3 reference accelerometers) have been used to conduct the experimental characterization. Furthermore, the duration of the data collection conducted in each setup may be often very brief. A roving instrumentation scheme is generally used instead of an instrumentation scheme in which all accelerometers remain at stationary locations on the structure throughout the testing program for the following reasons: (1) limited numbers of available sensors or data acquisition channels, (2) the capability to rapidly characterize the structure at many measurement stations, and (3) the ability to estimate mode shapes with a relatively fine spatial resolution. These reasons appear to be weighted more towards logistical considerations than towards possible impacts on the reliability of the resulting identification.

The instrumentation scheme developed for the test bridge incorporated multiple stationary reference locations with a number of roving accelerometers (**Fig. 2**). A total of forty five unidirectional accelerometers were used in this scheme to measure the vibration responses. Of this total, thirty accelerometers remained at stationary positions throughout the testing, and served as reference sensors. The remaining fifteen accelerometers were roved across the spans and towers of the bridge to measure the structural responses in these regions with added spatial resolution in four roving setups. This instrumentation scheme enabled multiple and spatially distributed reference sensors to be utilized with the vibration data analysis performed for each region of the bridge (side spans, main span, and towers). It was expected that having multiple and spatially distributed reference locations available for the data analysis and interpretation stages would enable a significantly more stable and reliable identification of the dynamic characteristics than would be possible from a conventional roving setup with only a few reference locations.

2.3.1 Reference Measurement Stations

The thirty reference accelerometers were positioned in a total of ten measurement stations that were located on the main and side spans, and in the north and south towers. Each tower contained two measurement stations, one station located at the top of the tower and one station located near the mid-height of the tower (roadway level). One reference station was located near the middle of the both the north and south side spans. The final four reference measurement stations were distributed on the main span (**Fig. 2**). A typical layout of the three accelerometers used at each reference measurement station for the side spans and the main span is illustrated in **Fig. 3a**. A vertically oriented accelerometer was installed at the lower panel points of the stiffening trusses on the east and west sides of the bridge. The two accelerometers enabled both vertical and torsional vibration responses of the stiffening truss to be

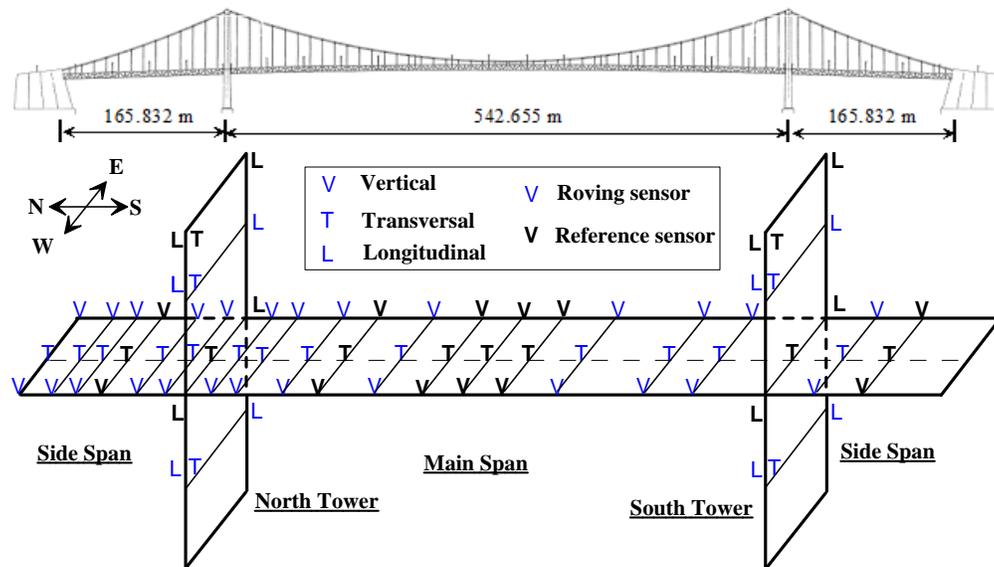


Fig.2 Accelerometer layout on bridge deck and tower

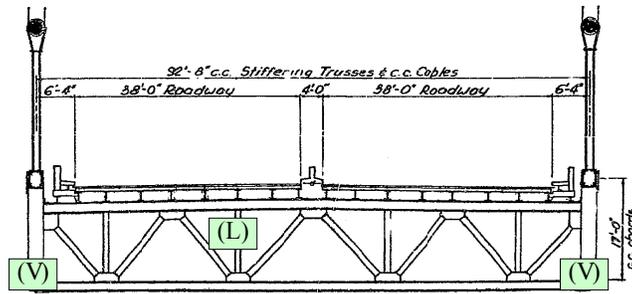
recorded at each measurement station. The third accelerometer at each span measurement station was installed laterally to record the lateral vibration responses (**Fig. 3b**). This lateral accelerometer was installed near the mid height of the floor truss as shown in **Fig. 3b** to reduce the effects of torsional vibrations on the measured lateral responses. The three accelerometers installed at each reference station were oriented in the same manner to measure vertical, lateral and longitudinal vibration responses of the structure. The stationary reference measurement stations enabled the vibration responses of the entire structure to be measured simultaneously and these measurements were linked together with the measurements recorded from the roving (local) accelerometers described in next section.

2.3.2 Roving Measurement Stations

A total of four roving setups were developed in order to provide added spatial resolution to the measurements recorded from the various sub-regions of the bridge (side spans, main spans, and towers) that contained the stationary reference accelerometers. In the first test setup, the fifteen roving accelerometers were deployed at five measurement stations located in the north side span. The second setup included five roving measurement stations distributed between the north tower, the north side span, and the main span. In the third test setup, a total of five roving measurement stations were distributed in the main span. The final roving setup included two measurement stations at different elevations in the south tower, and three measurement stations on the stiffening trusses in the south side span and the main span. **Fig. 2** illustrates the locations of all these roving accelerometers on the bridge. These roving accelerometers permitted a finer spatial resolution for the instrumentation layout in each sub-region of the bridge that was tested while also maintaining the capability to simultaneously measure the vibration responses of the entire structure.

2.4 Field Test and Data Collection

The field testing stage of the vibration testing program included both the installation and verification of the operation of the accelerometers, cabling, and data acquisition components. The accelerometers were installed on the steel bridge members at their designated locations using magnetic sensor mounts. The individual cables for each accelerometer were installed and temporarily secured



(a). Accelerometer placement on stiffening truss



(b). Installed lateral accelerometer

Fig. 3 Typical accelerometer layout on stiffening trusses at span measurement stations

along the bottom chord of the stiffening trusses. Careful execution of the instrument installation and on-site quality control may reduce the uncertainty by identifying the possible causes of error and their impact on the measurements. For instance, calibrating each sensor individually and the sensory system as a whole in the field environment may reduce the systematic and random errors in the experiment stage. A verification procedure was applied to each accelerometer before commencing the measurement procedure. This procedure involved manually oscillating each accelerometer in the direction of its sensitive axis and observing its response with an oscilloscope function on the data acquisition system. The expected response due to the input was that of a sine wave. In cases where errors were observed in the response of a given accelerometer, the continuity of the signal voltage between the data acquisition unit and the accelerometer were systematically checked at all connection locations (breakout boxes, accelerometer end, patch cables at the data acquisition enclosure, etc.). This process helped to identify and fix any malfunctioning sensors or cables.

After all the accelerometers and cables were verified, the data acquisition system was set to record one hour long data sets throughout the duration of the ambient vibration test program. The vibration measurements were sampled at 200 Hz which was at least 20 times greater than the estimated maximum frequency of interest. At the end of each one hour measurement period, the data were written to the internal hard drive of the controlling laptop computer. Multiple data sets were recorded in this manner from each of the four test setups.

3. DATA PRE-PROCESSING FOR UNCERTAINTY REDUCTION

Errors in measured structural response can produce inaccurate and unreliable identification results and if not mitigated, they will render any decisions made using the results of the identification incorrect. For instance, a typical data segment with poor quality is shown in **Fig. 4a**. The plot of the power spectral density function (PSD) shown in **Fig. 4c** has a significantly different character with that of a PSD

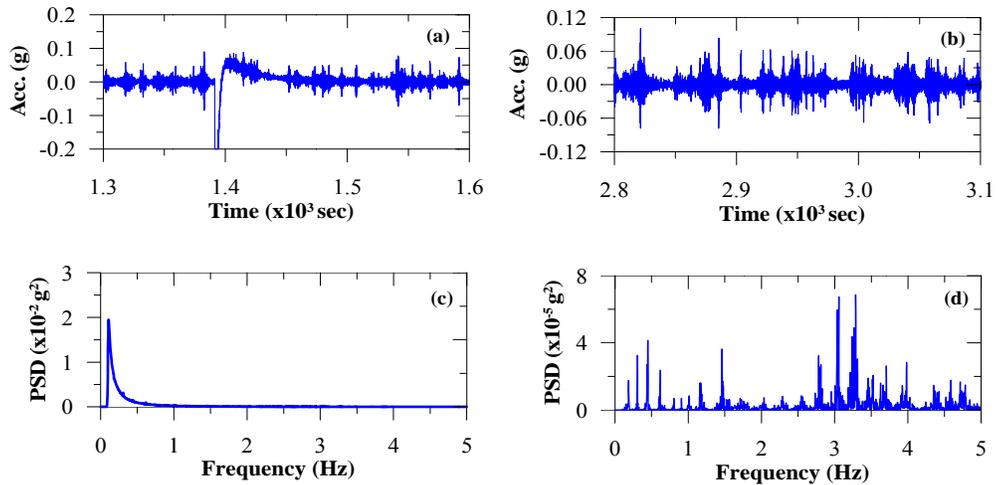


Fig.4 Records with poor and good data quality, and their PSD functions

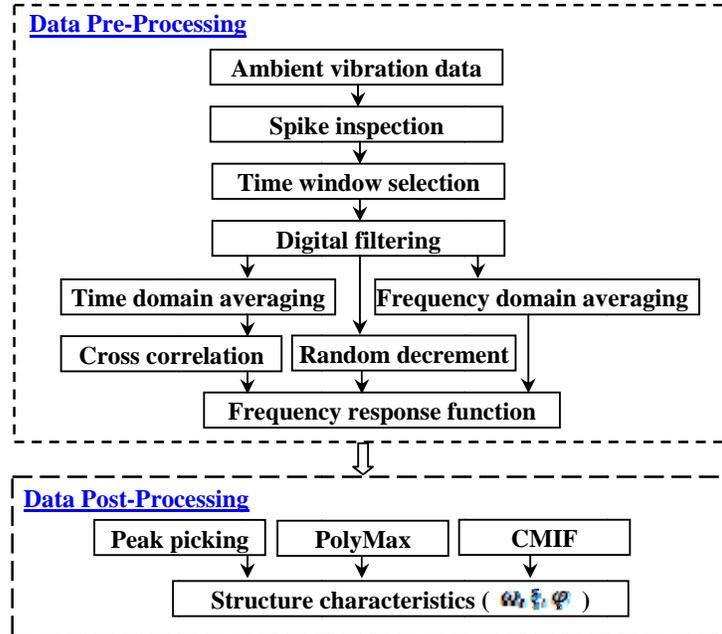


Fig.5 Flowchart of the proposed load protocol development procedure

function (**Fig. 4d**) of another time segment containing good data (**Fig. 4b**). This type of unreliable data is common in the raw data measured from constructed systems with intense electro-magnetic interference. It is essential to identify and mitigate erroneous data in order to extract reliable results from experiments. Due to the presence of such errors in the data measured from the bridge the data were conditioned before proceeding to modal parameter estimation. An automated data pre-processing procedure was developed and implemented and is illustrated in **Fig. 5**, which includes three steps as presented below.

3.1 Initial Data Processing

The first step to ensuring high quality data involved de-trending the measured acceleration records. This process was performed using MATLAB and enabled the removal of any linear DC offset. These acceleration records were then visually inspected by plotting their time histories to identify any malfunctioning sensors. If an accelerometer contained repetitive pronounced errors such as bias or large spikes (**Fig. 4a**), the channel was tagged and disregarded from further processing and analysis. Subsequently, a time window selection algorithm was developed to automatically remove time windows with pronounced noise within selected acceleration records. This involved segmenting the entire data record into a series of time windows containing 512 data points, and computing the mean and standard deviation of the acceleration amplitude for each time window. If the standard deviation for a given window was found to be large, that section of data was tagged for closer visual inspection. After the time window selection analysis was completed for the entire data record, a digital Butterworth band pass filter with cut-off frequencies at 0.1 and 8 Hz was designed to reduce the low and high frequency components embedded in the data that would adversely affect further data processing.

3.2 Construction of Cross Correlation Functions

Transformation of ambient vibration data to free-decay structural responses through correlation of time records is a popular time domain preprocessing technique. In the case of ambient vibration records, the cross correlation is the relationship between two acceleration channels at different locations in time. An estimate of the cross correlation function of $x(t)$ and $y(t)$, where $x(t) = x(i\Delta t)$ and $y(t) = y(i\Delta t)$ for $i = 1, 2, \dots, N$, is given by the following expression where $m < N$

$$\hat{R}_{xy}(i) = \frac{1}{N-i} \sum_{j=1}^{N-j} x_j y_{j+1} \quad i = 1, 2, \dots, m \quad (1)$$

where i is defined as the lag number and m is the maximum number of lags. James et al. (1995) proved that the cross-correlation function is indeed a sum of decaying sinusoids with the same characteristics as the impulse response functions of the original system, thus the cross-correlation function can be used for time-domain modal parameter estimation schemes. The random decrement (RD) method is an alternative approach to constructing the free decay structural response of a system as documented by Ibrahim (1977) and Zhang et al. (2009).

The Fast Fourier Transform (FFT) was used to transform the free decay responses to the frequency domain. The transformation to the frequency domain produced the crosspower spectral densities (CSD) which could be used with frequency domain modal parameter estimation techniques. When an incomplete time history is transformed by the FFT algorithm, it will result in errors affecting both amplitude and frequency parameters. Therefore, an exponential window was applied to the correlation functions to help minimize the effects of leakage. An exponential window was applied in the time domain by multiplying the signal by a time varying exponential function. In this study, the exponential function was designed to reduce the amplitude at the end of the signal to 0.001% of the initial amplitude of the signal. **Fig. 6a** shows a random vibration signal through the cross correlation method and its windowed form. The crosspower spectral densities obtained from the vibration data with and without exponential window processing are illustrated in **Fig. 6b**. It is observed that both curves show peaks at the same frequencies, however, the curve with the exponential window is smoother and the peaks are easier to identify.

3.3 Data Averaging in the Time Domain or Frequency Domain

Uncertainties in the St-Id process are generally separated into two categories: (1) epistemic or systematic uncertainty and (2) aleatory or statistical uncertainty. Systematic errors, often referred to as bias errors, generally are associated with electro-magnetic interference, effects of wind on cables and connections, and many other possible sources. Random errors are usually assumed to be distributed according to the normal law of errors, i.e., the error value as a random variable follows a Gaussian distribution. Data averaging is an efficient way to reduce random noise and its effects on structural identification.

There are two general methods employed for averaging: time domain averaging and frequency domain averaging. In the time domain method, a complete acceleration time record was first divided into a number of windows having the same length, and each segment was transferred to a free-decay random vibration data time history by using the cross correlation method coupled with exponential windowing. Averaging these free-decay random vibration data time histories of each segment would provide a time histories for further data processing. In contrast, Frequency domain averaging involves obtaining CSD estimates through the Fourier transform of all time segments and then averaging these estimates to produce an averaged CSD estimate. Given the significant effect of uncertainty on a single estimate of the CSD, averaging of repeatedly observed CSD estimates would greatly enhance the overall estimation of the CSD. To compare the effects of these two averaging methods, **Fig. 7a** illustrates the CSD estimates from both the time domain averaging and the frequency domain averaging. The singular value decompositions (SVD) of the CSD matrices from these two averaging methods were performed and the Single Value (SV) magnitudes are plotted in **Fig. 7b**. It should be noted that peak values of the SV magnitudes correspond to the structural frequencies, which will be introduced later in the CMIF post-processing method. It is observed that both averaging methods yield very similar frequency results.

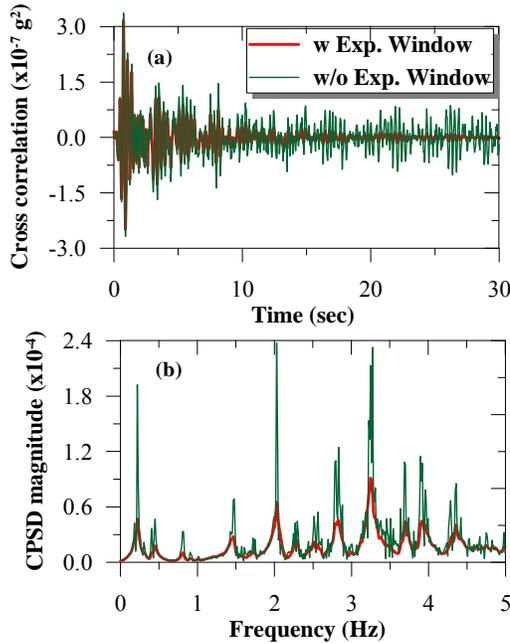


Fig.6 Construction of cross correlation function and utilization of exponential window

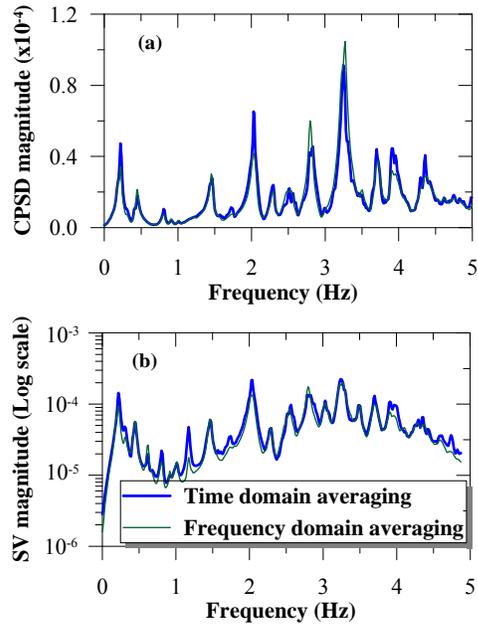


Fig.7 Time domain and frequency domain averaging

3.4 Data Relevance and Reliability Evaluation

Providing reliable information about the operational status of structures is critical to manage, update, and repair structures in an effective and efficient manner. A data pre-processing procedure has been proposed to reduce various errors involved in the test data. Before the cleaned data are used for parameter identification by post-processing methods, a data relevance and reliability evaluation are necessary for guaranteeing the quality and reliability of the information on which decisions are founded. For this purpose, an acceleration time history (Fig. 8a) observed on the main span was divided into two parts, data A and data B. Then they were pre-processed respectively by using the proposed procedure

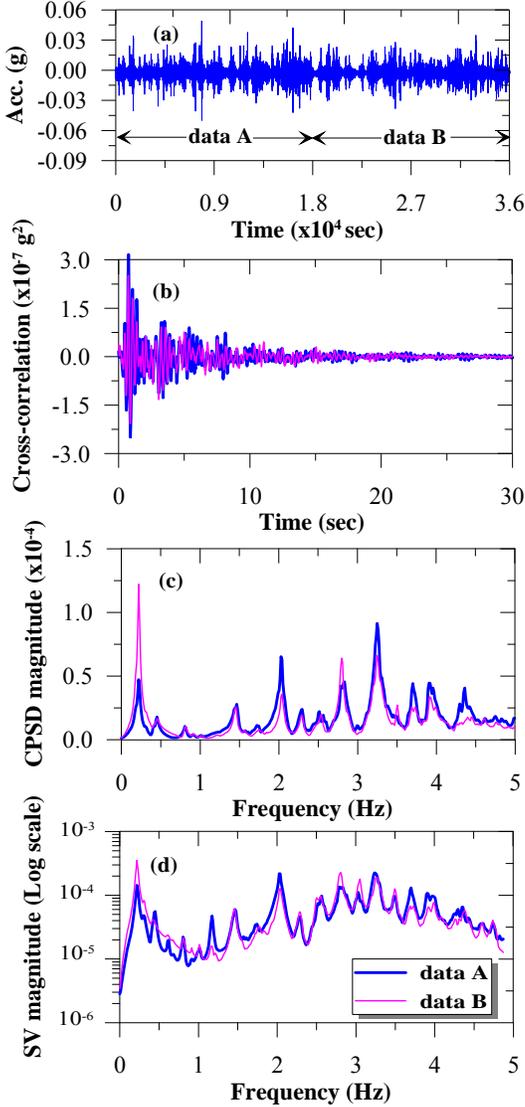


Fig.8 Data relevance and reliability evaluation

including time window selection, data filtering, cross-correlation construction, and exponential windowing. The calculated random vibration data from data A and data B are plotted in **Fig. 8b** for comparison. To further study the structural characteristics involved in these two data sets, CSD estimation from these two data sets are plotted in **Fig. 8c**. It is clear that structural frequencies corresponding with the peaks from these two curves agree very well. This means that different time segments with error reduction processing can produce similar modal parameter identification results and demonstrate that data A and data B have a good relevance and they are reliable for inclusion in the structural identification process. **Fig. 8d** plots the SV magnitude of the CSD matrix versus frequency generated by using data A and data B, from which the same conclusion, as described previously, can be drawn. The data relevance and reliability evaluation procedure described above verifies that the pre-processed data are of good quality and are sufficient for reliable data post-processing.

4. DATA PROCESSING

After the raw data obtained from the ambient vibration test are pre-processed to reduce uncertainty and ensure data quality, the Peak Picking, PolyMax, and Complex Mode Indicator Function (CMIF) methods are applied in the post processing stage to identify the suspension bridge dynamic characteristics.

4.1 Peak Picking Method

Peak Picking is one of the most basic approaches used in identifying structural modal properties from the output only measurements recorded in an ambient vibration test. The basic premise of this approach is that when a lightly damped structure is subjected to random excitation, the power spectral density (PSD) of channels will reach a maximum at the structural resonant frequencies and excitation spectrum peaks. Spurious noise or errors in the measurement may also lead peaks in the output spectrum and may increase the difficulty in obtaining accurate modal parameters. Several characteristics exist to help distinguish between the output spectral peaks that are due to structural modes, excitation spectrum peaks, or other noise. The first characteristic is that all points on a structure responding in a lightly damped normal mode of vibration will either be in phase or 180 degrees out of phase with one another. Therefore, phase angle of the spectra can be used to separate real modes from spurious ones. The phase between any pair of output measurements can be determined from the cross spectrum estimated between them. Ordinary coherence functions can also be used to help identify the peaks associated with the normal mode frequencies. The coherence functions tend to peak at the normal modes since the normal modes appear as narrow band peaks in the output spectra and the signal-to-noise ratio in the calculations is maximized at these frequencies. The normal mode shapes associated with the resonant frequencies are estimated from the responses of sensors distributed on the structure by the following expression (Bendat and Piersol 1980):

$$\varphi_i = (G_{jj}(\omega_i))^{1/2} \quad (2)$$

where $G_{jj}(\omega_i)$ denotes the output autospectral density value at the i^{th} normal mode frequency and the j^{th} location. The vibration responses at all measurement station locations utilized for each test setup (including the stationary reference locations) were simultaneously recorded during the ambient vibration testing of the long span suspension bridge. This permitted each measurement station to be considered as an output channel and also an independent reference channel during data processing by Peak Picking. In other words, each output location was compared relative to all other output locations in the same measurement direction acting as reference channels. The resulting "multiple reference" mode shapes were expected to provide additional insight regarding the consistency and reliability of the identified peak frequencies and would help in removing any noise-related or otherwise unreliable natural frequencies from the final identification results.

4.2 PolyMAX Method

The modal parameter estimation was also performed independently using the PolyMax method in the LMS software environment. This method is an adaptation of the Least Squares Complex Frequency

(LSCF) domain estimation algorithm. The Polymax method assumes the FRFs or crosspower spectral densities take the form shown below:

$$H(\omega) = \sum_{i=0}^p z^i \beta_i \cdot \left(\sum_{i=0}^p z^i \alpha_i \right)^{-1} \quad (3)$$

where $H(\omega)$ is the matrix containing all FRFs, β_i is the numerator matrix polynomial coefficients, α_i is the denominator matrix polynomial coefficients, and p is the model order. A z-domain model, $z = e^{-j\omega\Delta t}$, is used to model the FRFs in equation (3), where Δt is the sampling time. A z-domain model is a frequency domain model that is constructed from a discrete time model as opposed to a Laplace formulation that is constructed from a continuous time model. Eq. (3) can be written at every frequency line and then solved using a least squares solution. The resulting values are the matrix coefficients β_i and α_i . After these coefficients are calculated, the eigenfrequencies, damping ratios, and modal participation factors can be solved through the following equation:

$$\begin{pmatrix} 0 & I & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & I \\ -\alpha_0^T & -\alpha_1^T & \cdots & -\alpha_{p-2}^T & -\alpha_{p-1}^T \end{pmatrix} \cdot V = V\Lambda \quad (4)$$

where V contains modal participation factors, Λ contains the poles $e^{\lambda_i \Delta t}$ on its diagonal, and λ_i (λ_i^*) = $-\xi_i \omega_i \pm j \sqrt{1 - \xi_i^2} \omega_i$. After structural frequencies and damping ratios are obtained from the above equation, the mode shapes are calculated by solving the following equation using a least squares approach:

$$H(\omega) = \sum_{i=1}^n \frac{\varphi_i l_i}{j\omega - \lambda_i} + \frac{\varphi_i l_i^*}{j\omega - \lambda_i^*} - \frac{LR}{\omega^2} + UR \quad (5)$$

where n is the number of modes, φ_i is the mode shapes, l_i is the modal participation factors, and l_i^* denotes complex conjugate transpose of l_i . Matrix LR and UR are the lower and upper residuals respectively, which are terms that take into account the effect of out of frequency band modes on the modes contained inside the frequency band of interest.

4.3 Complex Mode Indicator Function (CMIF) Method

The CMIF method is also applied for the characterization of the suspension bridge by using the ambient test data. It is a spatial domain method in which the eigenvalues and eigenvectors are estimated directly by using Singular Values Decomposition (SVD) of the measured FRF or CPSD matrix. It can be shown that the general equation representing the connection between the measured FRF or CPSD matrix and the modal parameters is as follows:

$$H(\omega_i) = \psi(\omega_i) \cdot \frac{1}{j\omega_i - \lambda_i} \cdot L(\omega_i) \quad (6)$$

where ψ is the modal vector, L is the modal participation matrix, λ_i is the eigenvalue of mode i . The SVD of the FRF or CPSD matrix at a spectral line ω_i can be computed from the following equation:

$$H(\omega_i) = U(\omega_i) \cdot S(\omega_i) \cdot V(\omega_i) \quad (7)$$

where S is the diagonal singular value matrix, U and V are left and right singular vectors, respectively. It is seen from Eqs. (6) and (7) that system pole λ_i and input frequency ω_i are closer along the frequency line near a resonance, which results in a local maximum of the Singular Values (SV) magnitude. A plot of the SV magnitude of the FRF or CSD matrix (CMIF plot) is able to determine the location and number of eigenvalues in a data set. The left singular vector associated with the peak singular value is the approximate modal vector of the system. Enhanced frequency response function in the modal space can be produced using the dominant left singular vectors to generate and produce more accurate estimates of structural frequencies and damping ratios. The CMIF method can be considered as an SVD extension of the Peak Picking method. Practical applications of the CMIF method have illustrated that the CMIF approach is a very successful algorithm to identify modal parameters reliably,

even when actual structures having much more complex FRF characteristics, such as time variance due to ambient conditions (Catbas et al. 2004).

5. STRUCTURAL IDENTIFICATION RESULTS

5.1 Span Results

The vertical, longitudinal, and lateral responses were observed directly during the ambient vibration test from measurement stations on the suspension bridge spans. The torsional component of the vibration responses was calculated by taking the difference between the responses from the vertical accelerometers located on the east and west sides of a span or a tower. As described earlier, a pseudo-roving instrumentation scheme was designed to concentrate accelerometers in sub-regions of the bridge (north side span, south side span, main span, north tower, and south tower). These local measurements, together with the measurements from the large number of reference channels, were used to perform the structural characterization. For instance, the decoupled vertical responses from the roving and stationary accelerometers at measurement stations on the main span were utilized to identify frequencies and modal shapes of the main span in the vertical direction. After the initial data pre-processing including visual inspection, time window selection and frequency band pass filtering, the cleaned data were averaged with a 50% overlap of adjacent data blocks in the time domain to reduce random error, and the cross correlation functions of the averaged acceleration time histories were performed to generate the free-decay random vibration response. Because the conducted ambient vibration test was a multi-reference test, random vibration responses were produced by using each channel stationary measurement as the reference. The autopower spectral densities (ASD) were estimated for each response channel using Welch's method with a blocksize of 16,384 points. Crosspower spectral densities (CSD) were also estimated between each output channel and reference output channels. Because the sampling rate for the raw measurement data was 200 Hz, the selected blocksize yielded a frequency resolution of 0.0122 Hz. This was deemed adequate for identifying any closely spaced modes.

In the Peak Picking method, a mode indicator function (MIF) was first developed by using the normalized sum of the autopower spectral densities for a given component of the bridge in a given direction (Fig. 9a). The peaks in the MIF functions were selected using an automated peak identification routine. This routine compared the MIF magnitude at each frequency line with the values at the two frequency lines immediately before and after it. If the magnitude at the frequency line under consideration was larger than the others, it was selected as a possible peak. The MIF magnitude and normalized mode shape vectors corresponding to each identified peak were extracted from the matrix

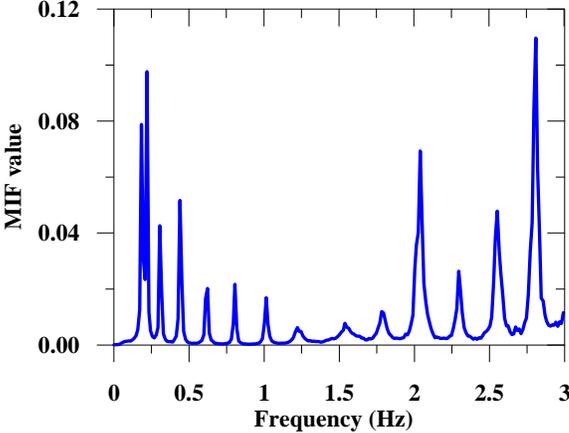


Fig. 9a Mode indicator function from the Peak Picking method

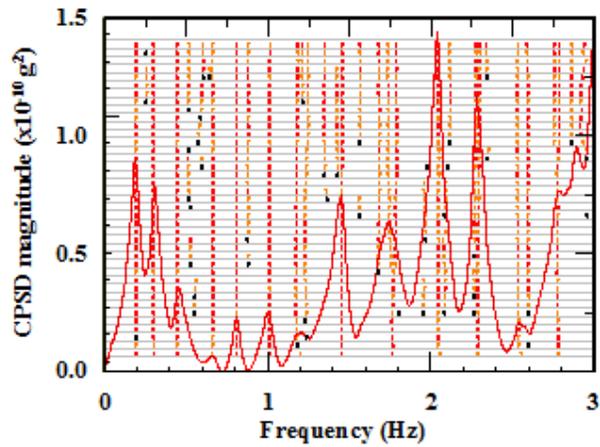


Fig. 9b Stabilization diagram from the PolyMax method

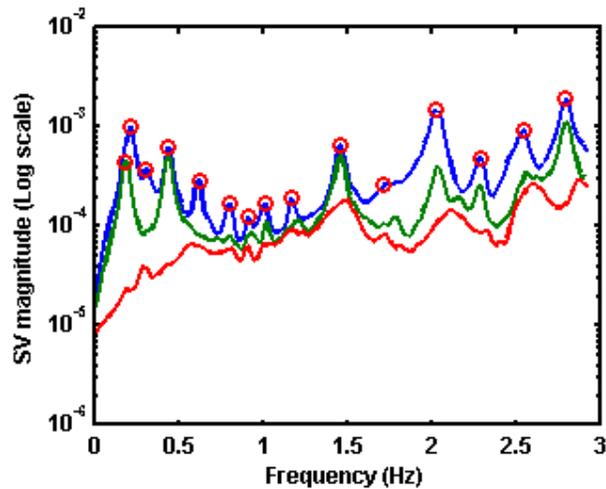


Fig. 9c Complex mode indicator function (CMIF) plot

containing these values at each frequency line. The identified peaks and their associated mode shapes were further scrutinized in an attempt to filter out any results that could be considered unreliable. Unreliable results can be associated with peaks in the excitation spectra, noise or errors in the frequency spectra, or by interactions between the oscillations of different bridge components. The preliminary mode shapes were inspected and the peaks associated with modes shapes that were essentially the same for all possible reference locations considered were deemed likely to be associated with the largest signal-to-noise ratios in the measurements. This procedure helped to rule out questionable peaks and mode shapes from the preliminary results. The dynamic characteristics of main span and other structural components in lateral, vertical, and transverse directions were identified from the Peak Picking method. For brevity purposes, only the main span frequencies identified in the vertical direction in the DC to 3Hz band are included in Table 1.

Similarly, the PolyMax method and the CMIF method were utilized to independently identify dynamic characteristics of the suspension bridge, and the identified frequencies of the entire span in the vertical direction from these two methods are provided in Table 1 to compare with those from the Peak Picking method. The major benefit of the PolyMax modal parameter estimation technique is in its ability to produce very clear stabilization diagrams. **Fig. 9b** shows the stabilization diagram obtained with the PolyMax method, from which all physical poles are clearly found and frequency and damping ratios are calculated. **Fig. 9c** shows a typical CMIF plot for the main span identification, from which structural frequencies are clearly identified at the CMIF plot peaks. To study the relevance and reliability of the collected data, the CMIF method was applied to data windows in order to extract several estimates of the structures modal parameters. The acceleration time records were divided into 34 time windows with a 50% overlap and a length of 40960 in the time domain. Then data in each time window was transformed to free-decay vibration response data using the cross correlation method and the corresponding CPSD matrix was generated using the Fourier transformation. Next, the CMIF method was utilized to identify the modal parameters in each time window. Statistical analyses of the multiple modal parameter estimates, including the mean and standard deviation of identified frequencies, are shown in **Table 1**.

Table 1: Identified main span vertical and torsional mode frequencies

Mode number	Peak picking	PolyMAX	CMIF (whole data)		CMIF (window by window)	
			(Time domain averaging)	(Freq. domain averaging)	Mean	St Dev
1	0.183	0.187	0.195	0.183	0.192	0.009
2	0.219	0.220	0.220	0.220	0.220	0.000
3	0.305	0.310	0.305	0.305	0.316	0.006
4	0.402	0.409				
5	0.439	0.444	0.452	0.439	0.439	0.004
6	0.476					
7	0.622	0.612	0.623	0.623	0.610	0.000
8	0.659	0.665	0.684		0.677	0.012
9	0.805	0.808	0.806	0.806	0.806	0.000
10	0.915	0.911	0.916	0.916	0.912	0.012
11	1.013	1.014	1.013	1.013	1.017	0.012
12		1.171	1.172	1.172	1.172	0.000
13	1.220	1.218			1.231	0.013
14	1.464	1.456	1.465	1.465	1.464	0.004
15		1.538	1.575		1.550	0.018
16	1.733	1.724	1.746	1.721	1.725	0.015
17		1.782			1.777	0.016
18	2.038	2.039	2.026	2.026	2.040	0.012
19	2.197	2.212			2.204	0.033
20	2.294	2.292	2.283	2.295	2.294	0.010
21		2.504	2.515		2.500	0.015
22	2.551	2.552	2.564	2.551	2.554	0.012
23	2.807	2.807	2.783	2.796	2.804	0.009
24		2.890	2.832		2.897	0.026

A preliminary correlation analysis among frequencies identified in the three modal parameter estimation procedures was performed. The vertical/torsional frequencies shown in Table 1 demonstrate excellent correlation between the three post processing procedures. The differences between most of the identified frequencies are less than one percent. The agreement between the results from the different time windows also demonstrates that the collected data are of high quality. The first three vertical mode shapes of the main span and the side spans identified from the Peak Picking and the PolyMax methods

also compare well with each other as shown **Fig. 10**. Similar conclusions could also be made from the identified structural parameters in other directions, although they are not included here for brevity. Different test data sets were investigated independently by various post-processing methods (the Peak Picking, PolyMax, and CMIF methods) and the identification results but their identified results agree well, which illustrate the identified results are reliable. It should be noted that the first mode shape identified for the main span has two peaks, which is similar to the vertical shape found in the Golden Gate Bridge identification (Pakzad and Fenves 2009).

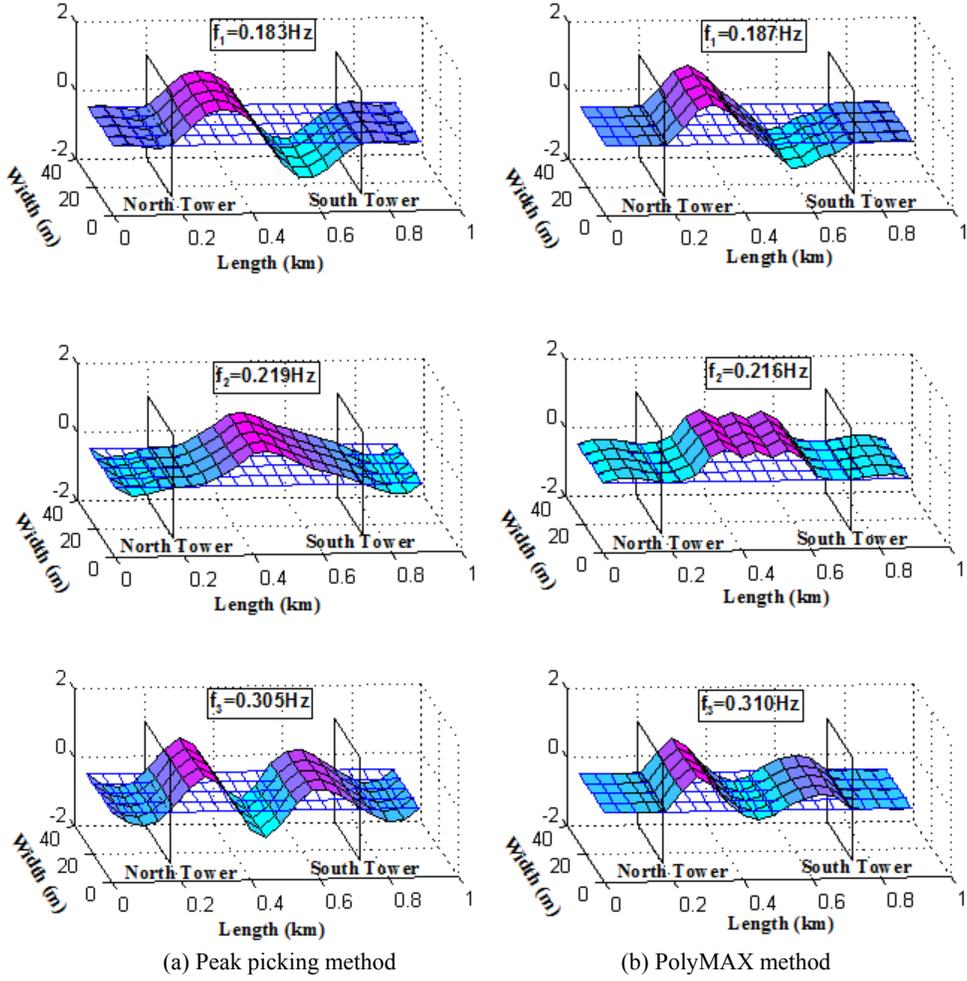
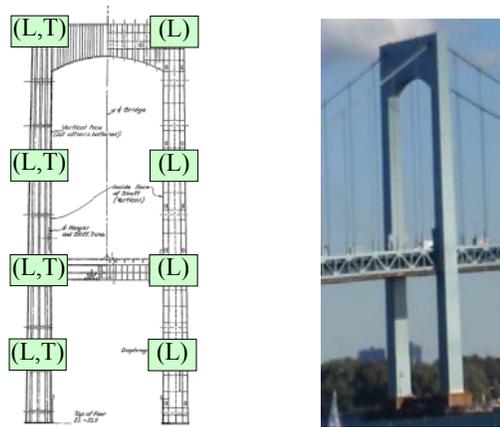


Fig. 10 Identified vertical shapes of the span

5.2 Tower Results

As described previously, the roving accelerometers in the second and fourth test setups together with reference accelerometers were installed on the north and south towers to measure their ambient responses. The four measurement stations at each tower during these test setups were located at different elevations along the height of the tower and each station included three accelerometers oriented in the longitudinal and transverse response directions as shown in **Fig. 11**. The same pre-processing procedure as that used for span response data was performed to clean the measured raw data of towers. The PolyMax method in the LMS software was used for tower response data post-processing. The identified tower frequencies are shown in Table 2, and selected mode shapes in the longitudinal direction are plotted in **Fig. 11**. It should be noted that there were some repeated tower mode shapes identified in the results. These shapes were generally not exactly the same at multiple frequencies, but the general character of the shapes was very similar. Further analysis would be required to establish if these repeated shapes are indicative of real responses, or if they the result of errors in the digital signal processing or interpretation of the measurement data.



(a) Instrumentation layout

(b) Tower photo

Fig.11 Typical accelerometer layout on tower

Table 2. Identified frequencies of the tower in longitudinal direction

Mode Number	Frequency (Hz)	Damping ratio (%)
1	0.378	2.15
2	1.849	3.30
3	1.888	2.04
4	2.035	2.55
5	2.547	2.47
6	3.017	1.23
7	3.285	1.70
8	3.502	0.29
9	3.644	0.29
10	3.916	0.73
11	4.281	1.10
12	4.734	1.22

Several challenges exist for reliable tower modal characterization by ambient vibration testing. One challenge is that the accelerations measured from the bridge towers generally have smaller amplitudes than those from spans due to the relatively stiffer tower structure, and the weak dynamic coupling between towers and spans. The tower response measurements will thus have a lower signal-to-noise ratio and more uncertainty than those from the spans. A second challenge is that the ambient dynamic excitation acting on the tower components of an in-service bridge can be expected to have a fundamentally different character from that which acts on the flexible spans. In ambient vibration testing, the dynamic excitation mainly due to traffic is transmitted from the spans to the towers. However, the spans of suspension bridges generally have relatively low frequencies (generally less than 5 Hz) and the dynamic excitation provided by traffic and wind acting on the spans is modified by the static and dynamic behavior of the spans themselves as it is transmitted to the towers. Furthermore, the towers will have a different bandwidth than the spans making it difficult for this modified excitation to resonate the towers. As a result, significant uncertainty exists in the ability to manipulate the ambient vibration response from the towers and extract reliable modal parameters. A forced-vibration test using a swept sine wave excitation from a small linear mass shaker was performed on one tower of this suspension bridge in order to verify the modal parameters identified from the ambient vibration test. The forced-vibration test data will be processed in the near future to provide additional estimates of the tower dynamic characteristics.

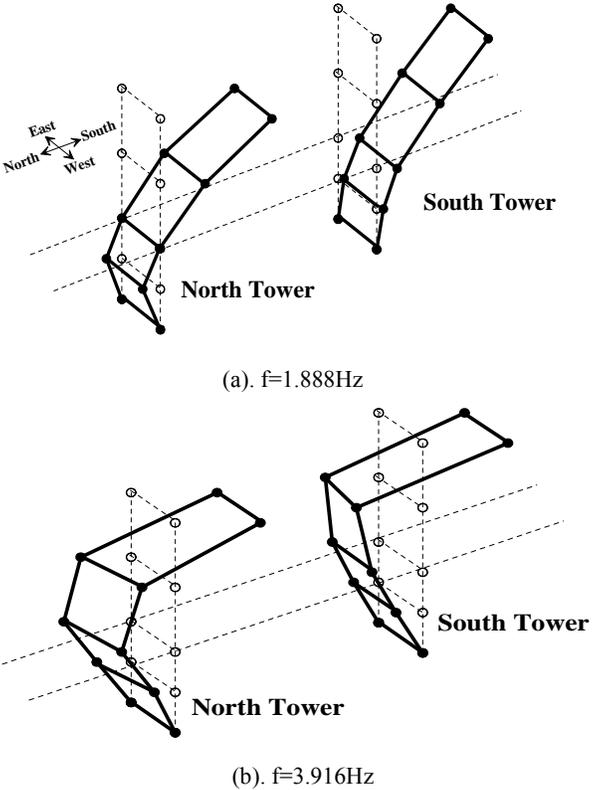


Fig.12 Tower mode shapes in longitudinal direction

6. CONCLUSION AND DISCUSSION

Ambient vibration test and data interpretation of a long-span suspension bridge has been presented, with a focus on mitigating various uncertainties in the experimental and data processing stages for reliable structural identification. The following conclusions can be drawn based on the investigation thus far:

(i) A multiple-reference testing scheme was developed and implemented to obtain high resolution vibration response measurements. Including multiple reference locations spatially distributed across the structure improved the effectiveness and reliability of the modal identification procedure. Multiple reference measurements also provided an effective means to identify the most likely candidates for the natural frequencies and mode shapes by visually comparing the consistency of the identified shapes constructed from different reference locations. Careful implementation of the experimental program in conjunction with diligent data quality control can effectively reduce uncertainty.

(ii) Various data pre-processing techniques have been developed to reduce data errors that are embedded in the raw field measurements. Visual inspection, time window selection, digital filtering, cross-correlation construction, windowing, and data averaging in time or frequency domain significantly reduce bias and random errors affecting the data quality and improve the reliability of the identified results.

(iii) Three data processing methods, including the Peak Picking, PolyMax, and CMIF methods, have been performed independently for modal parameter identification. Correlation analysis is used to verify the reliability of the identified results and provides a bridge owner more confidence in using the identified results for decision making.

Several areas require further investigation due to the complexity of the global St-Id problem and various uncertainties involved in each St-Id stage. A correlation between the identified modal parameters and the seismic vulnerability of the bridge needs to be addressed. The second area of continued work involves an accurate characterization of the dynamic properties of the tower structures. A low-amplitude forced excitation test was conducted on one tower using a portable shaker and multiple-reference analysis of the tested tower will be performed in the near future. The monitoring of the caissons is also a topic for future study because both the mass of the caisson and the mass of the tower will exhibit modal participation at high frequencies. Finally, whether there will be significant differences in the results from staged 2D versus simultaneous 3D processing of data will be explored.

Although the work presented in this paper did not include the analytical aspects of structural identification, it is important to note that an a-priori analytical model was developed and the predicted frequency and mode shapes were very helpful in evaluating the reliability of experimental results. It may appear ironic that one may benefit from analytically predicted results in processing experimental data. However, this is the most significant strategy a structural engineer may adopt. The benefits are due to the synergy in integrating analysis and experiment together with heuristics in understanding how complex constructed systems behave. Without predictive analysis and given the cloud of epistemic uncertainty governing the behavior of large constructed systems, it is very difficult to judge the reliability of a field experiment without the benefit of an expectation that is afforded by predictive analysis from an a-priori model. This issue will be further discussed in subsequent papers.

The final conclusion regards a need for advancing the operational model analysis of major constructed systems such as long-span cable-supported bridges. Significant mechanisms of uncertainty related to the coupling interactions between towers and spans, as well as the complexity of boundary, continuity and movement systems govern the structural behavior of such bridges. Many of these bridges in the US are aging, and there are concerns regarding changes in their reliability due to various aging and deterioration mechanisms. For example, the impacts of corrosion within the main cables of suspension bridges, and the impacts of vibration-induced stress amplifications at the anchorages of stay cables on the structural reliability are two major concerns. If the accuracy and reliability of operational modal analysis could be improved, it may prove to be an excellent means of monitoring the structural health and performance of major cable-supported bridges.

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